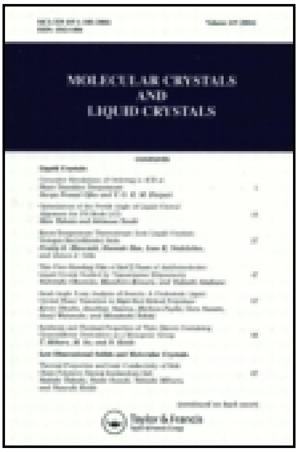

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Phase Singularity Birth Owing to Gaussian Beam Self-action in Nematic Liquid Crystal

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Phase Singularity Birth Owing to Gaussian Beam Self-action in Nematic Liquid Crystal

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ABSTRACT

We report on theoretical study of optical singularity birth in a wave front of light beam with astigmatic Gaussian profile passing through a homeotropically aligned nematic liquid crystal (NLC) cell. When a lineary polarised astigmatic Gaussian light beam illuminates the cell, the light induced Frederick's transition can be observed. Ritz's variational method is used, i.e. the profile of director reorientation is considered to be similar to astigmatic Gaussian profile. Parameters of the trial function are calculated numerically. The threshold intensity of light beam is appeared to increase with increasing of asymmetry of the beam under the constant value of laser beam area and cell thickness. Utilizing the Huygens-Fresnel principle and geometrical optics approximation we calculate the amplitude distribution in space for different distances from the cell. It is obtained that the trajectory of zero amplitude resembles a deformed rubber ring symmetrical in the *xz*-, *yz*-planes and stretched along z-axis.

Keywords: liquid crystal, phase singularities, light induced Frederiks transition.

1. INTRODUCTION

In early 70's, the structure of wave fronts in monochromatic wave was analysed in details by Nye and Berry[1] and by Wright[2]. It was shown that imperfections of regularity in optical wave front can occur even in a pure monochromatic wave. Natural birth of optical vortices is of principal interest. It was reported [3] the experimental study of the nucleation of wave front phase dislocations in a Gaussian beam experienced the self-action in a nematic liquid crystal.

Liquid crystals have high optical nonlinearity due to optically induced molecule reorientation. Because of this property self-action of incident on NLC Gaussian beam occurs. As a result, phase singularities appear. To investigate this process theoretically such problems should be solved: 1) to find equilibrium director profile under the action of incident light beam, 2) to calculate change of beam amplitude and phase with distance after the cell boundary. It is necessary to solve the Maxwell's equations for light propagation simultaneously with equations for LC reorientation. We study this problem approximately. First we find the liquid crystal director profile in a nematic cell illuminated with Gaussian light beam, neglecting the feedback. The light diffraction caused by the director inhomogeneity is considered after that.

2. PROFILE OF DIRECTOR DISTRIBUTION

Under the influence of external factors equilibrium state of a liquid crystal is characterized by the minimum of the free energy change:

$$\Delta F = F_d + F_m + F_e + F_S + F_l, \tag{1}$$

where F_d – elastic energy of a liquid crystal caused by deformation, F_e, F_m, F_l – terms, which appear under influence of an external electrical, magnetic and light field, F_S takes into account interaction between liquid crystal and cell walls.

Homeotropically aligned nematic liquid crystal cell is considered. Linearly polarised light with astigmatic Gaussian profile illuminates the cell:

$$\vec{E} = \vec{e}_x I^{\frac{1}{2}}, \ I = I_0 e^{-\frac{x^2}{R_1^2} - \frac{y^2}{R_2^2}}, \tag{2}$$

where $R_1 \neq R_2$.

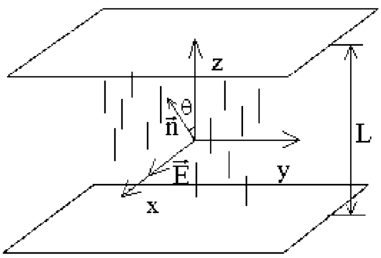

FIGURE 1 LC cell geometry, electrical field $\vec{E}$ and director $\vec{n}$ orientation

Oz axis of the Cartesian frame is directed along a non-perturbed direction of the director $\vec{n}_0 = (0,0,1)$, and *Ox* axis is directed along the polarisation of light (FIGURE1). Strong director anchoring at the cell walls is assumed.

The total free energy to be minimized consists of two parts: elastic energy associated with the director deformation in the cell volume and interaction between liquid crystal and light beam. For simplicity we employ one elastic constant approximation. Director reorientation occurs obviously in *XOZ* plane in that case. Director in a polar coordinate system is given by:

$$\vec{n} = (\sin\theta, 0, \cos\theta). \tag{3}$$

The total free energy now has the form:

$$F = \frac{K}{2} \int_V ((\theta_x')^2 + (\theta_y')^2 + (\theta_z')^2) dV - \frac{\varepsilon_a}{16\pi} \int_V \sin^2\theta \left|\vec{E}\right|^2 dV \tag{4}$$

where K is elastic constant, ε_a is the anisotropy of the LC dielectric tensor, V is the volume of the cell. $\sin^2\theta$ is taking Taylor up to forth power.

We use the Ritz's variational method to find the director profile under the action of Gaussian light beam and seek solution in the form:

$$\theta = \theta_0 \exp\left\{ -\frac{x^2}{a_1^2} - \frac{y^2}{a_2^2} \right\} \sin\left(\frac{\pi z}{L} \right), \tag{5}$$

where L is the cell thickness, a_1, a_2 and θ_0 are the parameters to be found. Substituting this expression (5) into the free energy functional (4) and integrating over x, y, z, we get function depending now on variables a_1, a_2 , θ_0:

$$F = \frac{\pi K L \theta_0^2}{8} \cdot \frac{a_2}{a_1} + \frac{\pi K L \theta_0^2}{8} \cdot \frac{a_1}{a_2} + \frac{\pi K L}{8}\left(\frac{\pi}{L}\right)^2 a_1 a_2 \theta_0^2 - \frac{\varepsilon_a L I_0}{32} \times \\ \times\left(\theta_0^2\left(\frac{1}{R_1^2}+\frac{2}{a_1^2}\right)^{-\frac{1}{2}}\left(\frac{1}{R_2^2}+\frac{2}{a_2^2}\right)^{-\frac{1}{2}} - \frac{\theta_0^4}{4}\left(\frac{1}{R_1^2}+\frac{4}{a_1^2}\right)^{-\frac{1}{2}}\left(\frac{1}{R_2^2}+\frac{4}{a_2^2}\right)^{-\frac{1}{2}}\right). \tag{6}$$

The intensity I_0 appears in it as the parameter.

To find threshold value I_{th} when director starts to reorient from the initial angle $\theta = 0$ we need to determine the condition, under which the coefficient at θ_0^2 in function (6) changes its sign:

$$\frac{\pi K L}{8} \cdot \frac{a_2}{a_1} + \frac{\pi K L}{8} \cdot \frac{a_1}{a_2} + \frac{\pi K L}{8}\left(\frac{\pi}{L}\right)^2 a_1 a_2 - \\ - \frac{\varepsilon_a L I_0}{32}\left(\frac{1}{R_1^2}+\frac{2}{a_1^2}\right)^{-\frac{1}{2}}\left(\frac{1}{R_2^2}+\frac{2}{a_2^2}\right)^{-\frac{1}{2}} = 0 \tag{7}$$

Let's express intensity I_0 from (7) as the function of parameters a_1 and a_2:

$$I_0 = \frac{4\pi K}{\varepsilon_a}\left(\frac{a_2}{a_1} + \frac{a_1}{a_2} + a_1 a_2 \frac{\pi^2}{L^2}\right)\left(\frac{1}{R_1^2}+\frac{2}{a_1^2}\right)^{\frac{1}{2}}\left(\frac{1}{R_2^2}+\frac{2}{a_2^2}\right)^{\frac{1}{2}} \tag{8}$$

The minimum of expression (8) with respect to parameters a_1 and a_2 determines the threshold value of intensity. It can be computed for certain values of parameters. We chose them to be as follows: L=20µm, R_1=9µm, R_2=11µm, λ=0.63µm. Threshold intensity increases with increasing of asymmetry of the beam under the constant value of laser beam area and cell thickness.

The necessary condition for minimum of the free energy function is that partial derivatives of F with respect to a_1, a_2, θ_0 are equal to zero simultaneously. This condition gives us system of three non-linear equations. In FIGURE 2 we plot a_1, a_2, θ_0 dependencies on the intensity of input light beam,which are calculated numerically.

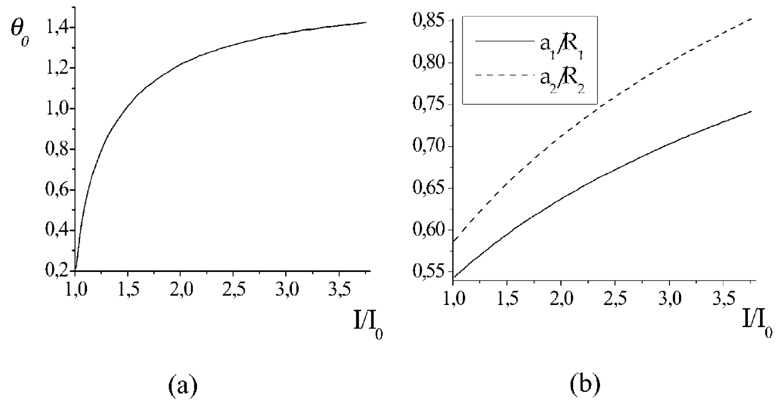

FIGURE 2 The parameters θ_0 (a) and a_1, a_2 (b) as the function of light beam intensity for R_1=9μm, R_2=11μm

3. WAVE PROPAGATION AFTER THE CELL

Appearance of wave front singularities in Gaussian beam passed liquid crystal cell, called optical vortices (OV), was observed experimentally in LC by Pishnyak et. al.[3] Phase defects appear at the points where wave amplitude vanishes on some line being axis of vortex [5]. Here the phase becomes undetermined. The slit of a wave front arises on a line of a dislocation. Around zero-amplitude, the crest of the wave transforms to the through.

Let's investigate now Gaussian light beam diffraction caused by the inhomogeneous director profile. In dielectrical, linear, isotropic, homogeneous medium without dispersion behaviour of all the component of electrical and magnetic field is described by one scalar wave equation [6]:

$$\Delta u(x,y,z,t) - \frac{n^2}{c^2} \cdot \frac{\partial^2 u(x,y,z,t)}{\partial t^2} = 0, \quad (8)$$

where

$$u(x,y,z,t)=\mathrm{Re}\big(U(x,y,z)\exp(-i\omega t)\big) \tag{9}$$

is any scalar field component. The dependence on time was allocated, only spatial part $U(x,y,z)$ will be examined below. To find light field behind LC cell the Huygens-Fresnel principle is used. In Fresnel's approximation, which is correct in a short-range zone, the principle looks like [6]:

$$U(x_1,y_1,z)=\frac{e^{ikz}}{i\lambda z}\iint_{\Sigma}U(x,y)\exp\left\{i\frac{k}{2z}\left[(x_1-x)^2+(y_1-y)^2\right]\right\}dxdy \tag{10}$$

where $U(x_1,y_1,z)$ is complex wave amplitude in the plane X_1OY_1, which position is at distance z from the initial plane XOY, $U(x,y)$ – beam complex amplitude in the initial plane $z=z_0$, k – wave vector, λ – wavelength. Equation (10) has no analitical solution in the common case. We provide therefore its numerical simulations for parameters typical for real experiments.

To calculate behaviour of the light field in our case it is necessary to know the light field at exit of cell, which is the initial for our problem ($z=z_0$). In a ray approximation this field has the form [7]:

$$U(x,y)=C\left|\vec{E}\right|^2\exp\left(i\frac{\omega}{c}\psi_1(x,y,L)\right) \tag{11}$$

where

$$\psi_1(x,y,L)=\int_0^L p_z dz' \tag{12}$$

is the phase retardation after passing through cell, and

$$p_z=\frac{n_o n_e}{\left(n_o^2\sin^2\theta+n_e^2\cos^2\theta\right)^{\frac{1}{2}}} \tag{13}$$

θ is deviation angle of the director, which we have found in the previous section (5). C in equation (11) is a constant, so we don't take it into account.

4. PHASE SINGULARITY BIRTH

Similar to the case of stigmatic Gaussian lens we expand now the expression (13) into series up to the 4th order of θ_0.:

$$p_z = n_o - \frac{1}{2} n_o \left(\left(\frac{n_o}{n_e} \right)^2 - 1 \right) \theta^2 + \frac{1}{24} n_o \left(\left(\frac{n_o}{n_e} \right)^2 - 1 \right) \left(9 \left(\frac{n_o}{n_e} \right)^2 - 5 \right) \theta^4, \quad (14)$$

where θ is deviation angle of the director, which we have found in the previous section (eq. (5)).

On integrating (14) over z and substituting the result into eq. (10), we get for complex amplitude:

$$U(x_1, y_1, z) \sim \iint e^{-\left(\frac{x^2 L^2}{R_1^2} + \frac{y^2 L^2}{R_2^2} \right)} e^{iG} \exp\left\{ i \frac{z_L}{z} \left[(x - x_1)^2 + (y - y_1)^2 \right] \right\} dxdy \quad (15)$$

where

$$\begin{aligned} G = {} & \frac{2\pi}{\lambda} n_o L - \frac{1}{4} \frac{2\pi}{\lambda} n_o L \left(\left(\frac{n_o}{n_e} \right)^2 - 1 \right) \theta_0^2 \exp\left(-2 \cdot \left(\frac{x^2}{x_{01}^2} + \frac{y^2}{x_{02}^2} \right) \right) \\ & + \frac{1}{64} \frac{2\pi}{\lambda} n_o L \left(\left(\frac{n_o}{n_e} \right)^2 - 1 \right) \left(9 \left(\frac{n_o}{n_e} \right)^2 - 5 \right) \theta_0^4 \exp\left(-4 \cdot \left(\frac{x^2}{x_{01}^2} + \frac{y^2}{x_{02}^2} \right) \right) \end{aligned} \quad (16)$$

Here we introduce new dimensionless variable $\tilde{x} = \frac{x}{L}$, $\tilde{y} = \frac{y}{L}$ and rewrite them as $\tilde{x} \equiv x$, $\tilde{y} \equiv y$, also $x_{01} = \frac{a_1}{L}$, $x_{02} = \frac{a_2}{L}$, $z_L = \frac{\pi L^2}{\lambda}$.

The amplitude distribution in space of the wave behind liquid crystal cell $U(x_1, y_1, z)$ is evaluated numerically for certain value of intensity ^{2}=1.13$^2_0$. The trajectory of zero amplitude resembles a deformed rubber ring symmetrical in the *xz*-, *yz*- planes and stretched along z-axis (FIGURE 3).

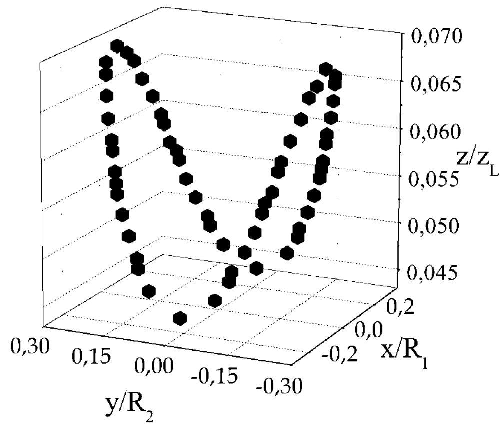

FIGURE 3. Trajectory of optical singularities in the astigmatic Gaussian beam after passing of NLC cell

In FIGURES 4,5 we plot numerically light wave amplitude and phase dependencies on x,y for different distances from the cell (here $z_L = \frac{\pi L^2}{\lambda}$). From FIGURES 4a, 5a one can seen that two point edge dislocation are born, dipole orients along y-direction. The phase ledge is seen in FIGURE 5a. Near the x-axis it has vertical wall of height π. Then the beam starts to diverge, four optical vortices appear and give rise to an "optical quadrupole" with alternating charges (FIGURE 4b,5b). FIGURE 5c shows usual helicoidal structure of optical vortice. There are two π-jumps near the dislocation point. Two wave fronts join here. The optical vortices centres approach to the x-axis. With increasing z another dipole can be seen but it orients along x-axis (FIGURE 4c,5c). Then intensity valleys become gray and optical singularities annihilate. The birth of this type of dislocation was examined by Kreminskaya et al. [8]. If one moves along z-axis the following optical dislocations can appear. Few of them can co-exist. There are two optical vortices at $z=0.692z_L$: one of them is near the point $x=0$, $y=0.25$, another is near $x=0.55$, $y=0.15$ (FIGURES 4c,5c).

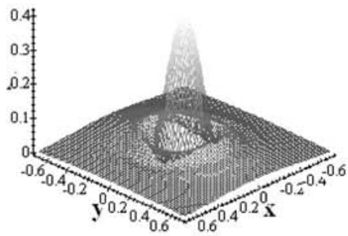

FIGURE 4a Distribution of the light beam amplitude at $z=0.0448z_L$

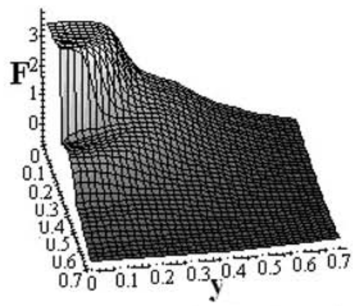

FIGURE5a Distribution of the light beam phase at $z=0.0448z_L$

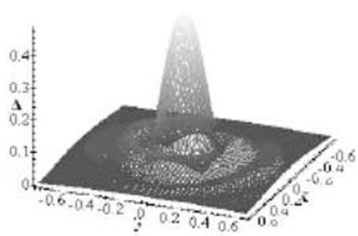

FIGURE 4b Distribution of the light beam amplitude at $z=0.057z_L$

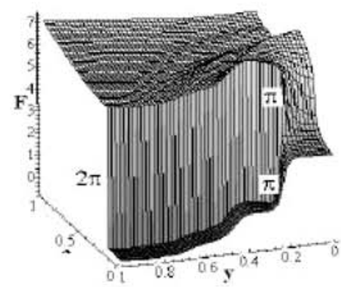

FIGURE 5b Distribution of the light beam phase at $z=0.057z_L$

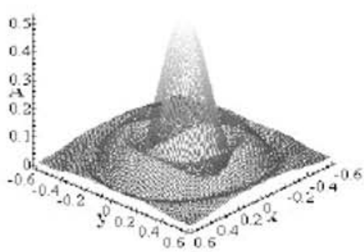

FIGURE 4c Distribution of the light beam amplitude $z=0.0692z_L$

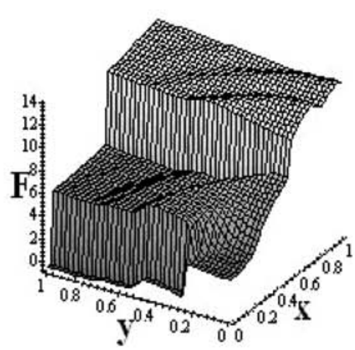

FIGURE 5c Distribution of the light beam phase at $z=0.0692z_L$

5. CONCLUSION

The director distribution profile in the cell with strong homeotropic anchoring under the action of astigmatic Gaussian light beam is founded in this paper. Director reorientation is threshold like. Threshold intensity increases with increasing of asymmetry of the beam under the constant value of laser beam area and cell thickness. We have found the conditions of the phase dislocations birth in coherent beam with initially smooth wave front using the obtained expressions for director profile. The trajectory of zero amplitude resembles a deformed rubber ring symmetrical in the *xz*-, *yz*- planes and stretched along z-axis. The borned quadrupole of the unity charged optical vortices moved in space during propagation along this trajectory.

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